Erratum

Volume **35**, No. 2 (1971), in the article "A Bifurcation problem for a Functional Differential Equation of Finitely Retarded Type," by N. Chafee, pp. 312–348, the example is incorrect, and in that which follows we shall point out the fallacy.

To construct the example we assumed the existence of a C^2-smooth function $g : [0, +\infty) \to [2, 3]$ and an infinite sequence $\{t_j\}_{j=0}^{+\infty}$ in $[0, +\infty)$ such that

$$\dot{g}(t) \to 0 \qquad \text{as} \quad t \to +\infty, \tag{1}$$

$$\ddot{g}(t)\, e^t \to 0 \qquad \text{as} \quad t \to +\infty, \tag{2}$$

$$0 = t_0 < t_1 < t_2 < \cdots, \tag{3}$$

$$t_j \to +\infty \qquad \text{as} \quad j \to +\infty, \tag{4}$$

$$g(t_{2j}) = 2 \qquad (j = 0, 1, 2,...), \tag{5}$$

$$g(t_{2j+1}) = 3 \qquad (j = 0, 1, 2,...). \tag{6}$$

The fallacy is that no such function and sequence exists.

Indeed, suppose that g: $[0, +\infty) \to [2, 3]$ is a C^2-smooth function satisfying (1) and (2). Then, there exists a constant $k > 0$ such that $|\ddot{g}(t)| \leqslant Ke^{-t}$ for all $t \geq 0$. Hence,

$$|\dot{g}(t)| \leq \int_t^{+\infty} |\ddot{g}(s)|\, ds \leq Ke^{-t} \qquad (0 \leq t < +\infty).$$

Now let $\{t_j\}_{j=0}^{+\infty}$ be any sequence in $[0, +\infty)$ satisfying (3) and (4). Then, for any integer $j \geq 0$, we have

$$|g(t_{2j+1}) - g(t_j)| \leq \int_{t_{2j}}^{t_{2j+1}} |\dot{g}(t)|\, dt \leq Ke^{-t_{2j}} - Ke^{-t_{2j+1}}.$$

Therefore, (5) and (6) cannot both be satisfied. Thus, we have our fallacy.

The example we are discussing was introduced in [1] in connection with Theorem 3.1 on pages 319–320. In that theorem we asserted the existence of certain ω-limit sets $\omega(\phi, \epsilon)$ each of which consists of one or more non-constant periodic orbits. The example was intended to show that a set $\omega(\phi, \epsilon)$ may contain more than one such orbit. In fact, however, the opposite is the case; each $\omega(\phi, \epsilon)$ contains no more than one nonconstant periodic orbit. This follows from a result obtained by N. Fenichel [2, Theorem 5].

It is Dr. Fenichel who has pointed out to us the fallacy described above and we are grateful to him for clearing up this entire matter.

References

1. N. Chafee, A bifurcation problem for a functional differential equation of finitely retarded type, *J. Math. Anal. Appl.* **35** (1971), 312–348.
2. N. Fenichel, Asymptotic stability with rate conditions, to appear.